

Nontrivial Relations between GPDs and TMDs

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- Introduction
- Nontrivial relation between GPD E and TMD f_{1T}^\perp
(Burkardt, 2002, ... / Burkardt, Hwang, 2003)
- Overview of model-dependent nontrivial relations
- GTMD analysis
- Summary

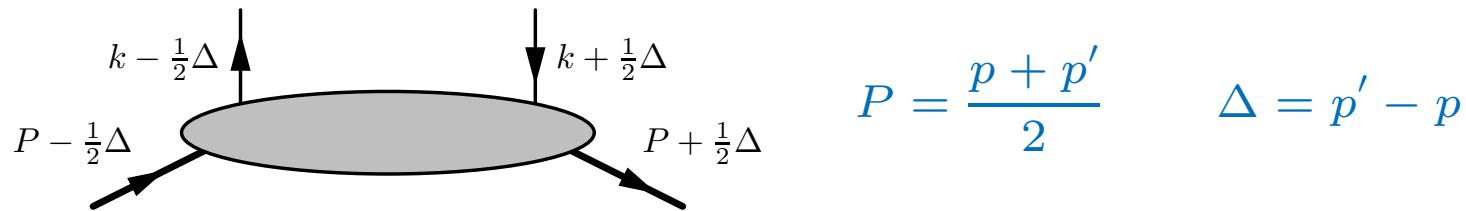
In collaboration with K. Goeke, S. Meißner, M. Schlegel
(hep-ph/0703176 ; arXiv:0805.3165 ; arXiv:0906.5323)

Definition of GPDs and TMDs

- GPDs

- Appear in QCD-description of hard exclusive reactions

- Kinematics



- GPD-correlator

$$\begin{aligned}
 F^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik \cdot z} \langle p'; \lambda' | \bar{\psi} \left(-\frac{z}{2} \right) \gamma^+ \mathcal{W}_{GPD} \psi \left(\frac{z}{2} \right) | p; \lambda \rangle \Big|_{z^+ = z_T = 0} \\
 &= \frac{1}{2P^+} \bar{u}(p', \lambda') \left(\gamma^+ H^q(x, \xi, t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E^q(x, \xi, t) \right) u(p, \lambda)
 \end{aligned}$$

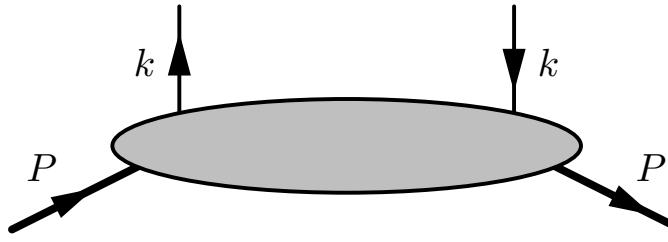
$$x = \frac{k^+}{P^+} \quad \xi = -\frac{\Delta^+}{2P^+} \quad t = \Delta^2$$

- Leading twist for

$$\bar{\psi} \gamma^+ \psi \quad \bar{\psi} \gamma^+ \gamma_5 \psi \quad \bar{\psi} i\sigma^{j+} \gamma_5 \psi$$

- TMDs

- Appear in QCD-description of hard semi-inclusive reactions
- Kinematics



- TMD-correlator

$$\begin{aligned}\Phi^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2 \vec{z}_T}{(2\pi)^2} e^{ik \cdot z} \langle P; S | \bar{\psi} \left(-\frac{z}{2} \right) \gamma^+ \mathcal{W}_{TMD} \psi \left(\frac{z}{2} \right) | P; S \rangle \Big|_{z^+=0} \\ &= f_1^q(x, \vec{k}_T^2) - \frac{\epsilon_T^{ij} k_T^i S_T^j}{M} f_{1T}^{\perp q}(x, \vec{k}_T^2)\end{aligned}$$

→ Sivers function f_{1T}^\perp induces (dipole) distortion of TMD correlator

→ New era: 3D-image of the nucleon in (x, k_T) -space

- Leading twist for

$$\bar{\psi} \gamma^+ \psi \quad \bar{\psi} \gamma^+ \gamma_5 \psi \quad \bar{\psi} i\sigma^{j+} \gamma_5 \psi$$

- Leading twist GPDs and TMDs

	Quarks			Gluons				
Forward	f_1^q	g_1^q	h_1^q	g	Δg			
k_T -dependent	f_1^q	$f_{1T}^{\perp q}$	g_{1L}^q	g_{1T}^q	f_1^g	$f_{1T}^{\perp g}$	g_{1L}^g	g_{1T}^g
	h_{1T}^q	$h_{1L}^{\perp q}$	$h_{1T}^{\perp q}$	$h_1^{\perp q}$	h_{1T}^g	$h_{1L}^{\perp g}$	$h_{1T}^{\perp g}$	$h_1^{\perp g}$
Generalized	H^q	E^q	\tilde{H}^q	\tilde{E}^q	H^g	E^g	\tilde{H}^g	\tilde{E}^g
	H_T^q	E_T^q	\tilde{H}_T^q	\tilde{E}_T^q	H_T^g	E_T^g	\tilde{H}_T^g	\tilde{E}_T^g

- Trivial relations:

$$H^q(\mathbf{x}, 0, 0) = f_1^q(\mathbf{x}) = \int d^2 \vec{k}_T f_1^q(\mathbf{x}, \vec{k}_T^2) \quad \text{etc.}$$

- Nontrivial relations: 3 for quarks, 4 for gluons
- Dipole pattern for $f_{1T}^{\perp q}$, $h_1^{\perp q}$, g_{1T}^q , $h_{1L}^{\perp q}$ Quadrupole pattern for $h_{1T}^{\perp q}$ → EIC !

Impact parameter representation of GPDs

- Fourier transform of GPD-correlator ($\xi = 0$) (Burkardt, 2000)

$$F^q(x, \vec{\Delta}_T, S) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik \cdot z} \langle p'; S | \bar{\psi}\left(\frac{z}{2}\right) \gamma^+ \mathcal{W}_{GPD} \psi\left(\frac{z}{2}\right) | p; S \rangle \Big|_{z^+ = z_T = 0}$$

$$\begin{aligned} \mathcal{F}^q(x, \vec{b}_T; S) &= \int \frac{d^2 \vec{\Delta}_T}{(2\pi)^2} e^{-i \vec{\Delta}_T \cdot \vec{b}_T} F^q(x, \vec{\Delta}_T; S) \\ &= \mathcal{H}^q(x, \vec{b}_T^2) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \left(\mathcal{E}^q(x, \vec{b}_T^2) \right)' \\ \text{with } \mathcal{H}^q(x, \vec{b}_T^2) &= \int \frac{d^2 \vec{\Delta}_T}{(2\pi)^2} e^{-i \vec{\Delta}_T \cdot \vec{b}_T} H^q(x, 0, -\vec{\Delta}_T^2) \end{aligned}$$

- Distortion of GPD-correlator in impact parameter space

$$d^{q,i} = \int dx \int d^2 \vec{b}_T b_T^i \mathcal{F}^q(x, \vec{b}_T; S) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \int dx E^q(x, 0, 0) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \kappa^q$$

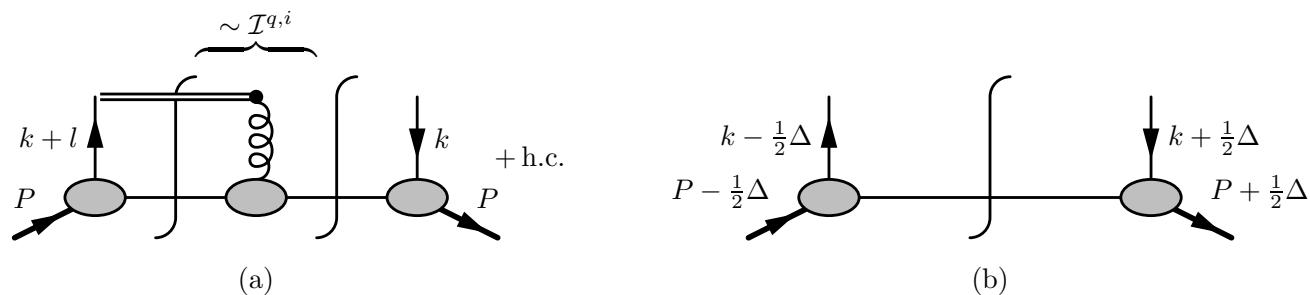
→ Flavor dipole moment of about 0.2 fm

- Relation between distortion and Sivers effect (Burkardt, 2002)
(Not obvious because QCD-description of, e.g., SIDIS does not know about GPD-correlator in b_T -space)
 - Quantitative nontrivial relation in spectator model (Burkardt, Hwang, 2003)

$$\begin{aligned} \langle k_T^{q,i}(x) \rangle_{UT} &= - \int d^2 \vec{k}_T k_T^i \frac{\epsilon_T^{jk} k_T^j S_T^k}{M} f_{1T}^{\perp q}(x, \vec{k}_T^2) \\ &= \int d^2 \vec{b}_T \mathcal{I}^{q,i}(x, \vec{b}_T) \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x, \vec{b}_T^2) \right)' \end{aligned}$$

- ### – Interpretation

Sivers effect = Distortion \otimes FSI

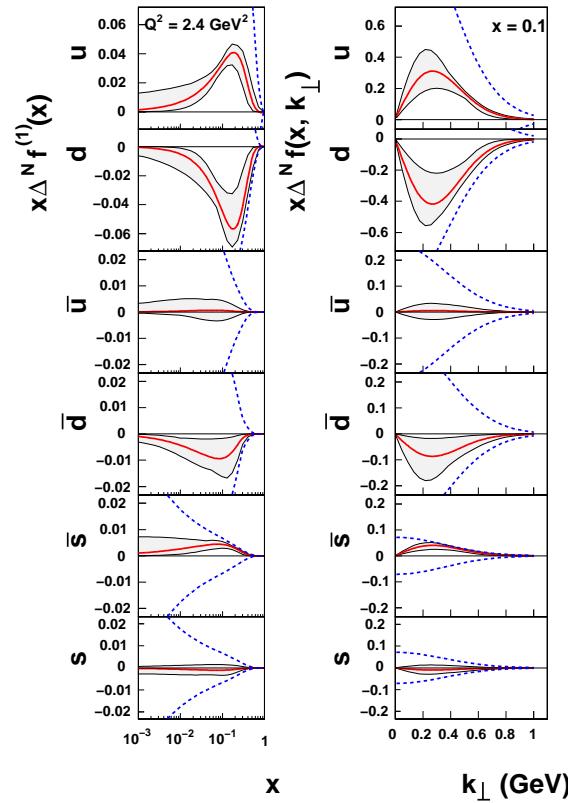


- Prediction

$$f_{1T}^{\perp u/p} \sim -0.8 f_{1T}^{\perp d/p} < 0$$

→ Relative (but not absolute) sign also from large N_c -analysis (Pobylitsa, 2003)

→ Extraction of $\Delta^N f = -\frac{2k_T}{M} f_{1T}^\perp$ by Anselmino et al., 2008



→ Nice agreement between qualitative picture and extraction

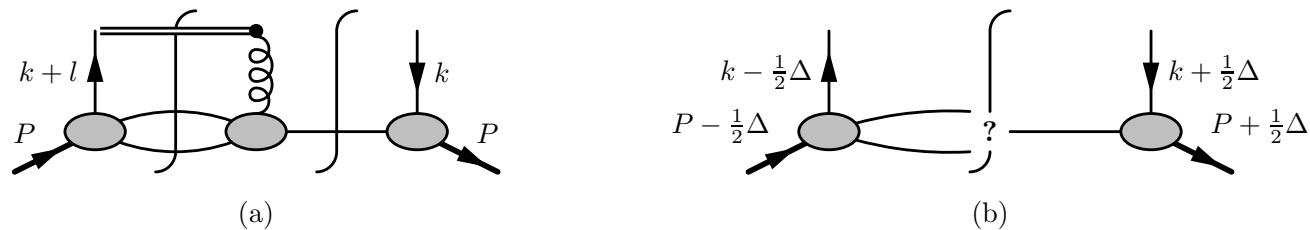
→ Agreement with qualitative picture and large N_c -analysis supports interpretation of HERMES signal for $A_{UT}^{\sin(\Phi - \Phi_S)}$ as Sivers effect

- Sign reversal of the Sivers function (Collins, 2002)

$$f_{1T}^\perp|_{\text{DY}} = -f_{1T}^\perp|_{\text{SIDIS}}$$

→ Sign reversal provided in qualitative picture:
lensing function \mathcal{I}^q changes sign (attractive vs repulsive interaction)

- Higher order contributions should spoil picture (Meißner, Metz, Goeke, 2007)



(confirmed by Gamberg, Schlegel, 2009)

- Spectator model: neglecting certain higher order graphs and introducing IR-regulator provides to all orders

Sivers effect = Distortion \otimes FSI

(Gamberg, Schlegel, 2009)

Comparing GPD- and TMD-correlator

- Additional relations by comparing the GPD-correlator with the TMD-correlator
(Diehl, Hägler, 2005)

$$\Phi^q(x, \vec{k}_T; S) = f_1^q(x, \vec{k}_T^2) - \frac{\epsilon_T^{ij} k_T^i S_T^j}{M} f_{1T}^{\perp q}(x, \vec{k}_T^2)$$

$$\mathcal{F}^q(x, \vec{b}_T; S) = \mathcal{H}^q(x, \vec{b}_T^2) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \left(\mathcal{E}^q(x, \vec{b}_T^2) \right)'$$

- Comparison allows one to find analogy:

$$f_{1T}^{\perp q} \leftrightarrow -\left(\mathcal{E}^q \right)'$$

- Comparison can be extended to other quark and gluon distributions

- No relation for GPDs \tilde{E} , \tilde{E}_T (drop out for $\xi = 0$) and TMDs g_{1T} , h_{1L}^\perp

- Relations of first type

$$f_1^{q/g} \leftrightarrow \mathcal{H}^{q/g} \quad g_{1L}^{q/g} \leftrightarrow \tilde{\mathcal{H}}^{q/g}$$

$$\left(h_{1T}^q + \frac{\vec{k}_T^2}{2M^2} h_{1T}^{\perp q} \right) \leftrightarrow \left(\mathcal{H}_T^q - \frac{\vec{b}_T^2}{M^2} \Delta \tilde{\mathcal{H}}_T^q \right)$$

- Relations of second type

$$f_{1T}^{\perp q/g} \leftrightarrow -\left(\mathcal{E}^{q/g}\right)' \quad h_1^{\perp q} \leftrightarrow -\left(\mathcal{E}_T^q + 2\tilde{\mathcal{H}}_T^q\right)'$$

$$\left(h_{1T}^g + \frac{\vec{k}_T^2}{2M^2} h_{1T}^{\perp g} \right) \leftrightarrow -2\left(\mathcal{H}_T^g - \frac{\vec{b}_T^2}{M^2} \Delta \tilde{\mathcal{H}}_T^g \right)'$$

- Relations of third type

$$h_{1T}^{\perp g} \leftrightarrow 2\left(\tilde{\mathcal{H}}_T^g\right)'' \quad h_1^{\perp g} \leftrightarrow 2\left(\mathcal{E}_T^g + 2\tilde{\mathcal{H}}_T^g\right)''$$

- Relation of fourth type

$$h_{1T}^{\perp g} \leftrightarrow -4\left(\tilde{\mathcal{H}}_T^g\right)'''$$

- Some consequences

- Relation for Boer-Mulders function $h_1^{\perp q}$ expected to match with the one for $f_{1T}^{\perp q}$

$$\begin{aligned}\langle k_T^{q,i}(x) \rangle_{UT} &= - \int d^2 \vec{k}_T k_T^i \frac{\epsilon_T^{jk} k_T^j S_T^k}{M} f_{1T}^{\perp q}(x, \vec{k}_T^2) \\ &= \int d^2 \vec{b}_T \mathcal{I}^{q,i}(x, \vec{b}_T) \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x, \vec{b}_T^2) \right)'\end{aligned}$$

$$\begin{aligned}\langle k_T^{q,i}(x) \rangle_{TU}^j &= - \int d^2 \vec{k}_T k_T^i \frac{\epsilon_T^{kj} k_T^k}{M} h_1^{\perp q}(x, \vec{k}_T^2) \\ &= \int d^2 \vec{b}_T \mathcal{I}^{q,i}(x, \vec{b}_T) \frac{\epsilon_T^{kj} b_T^k}{M} \left(\mathcal{E}_T^q(x, \vec{b}_T^2) + 2\tilde{\mathcal{H}}_T^q(x, \vec{b}_T^2) \right)'\end{aligned}$$

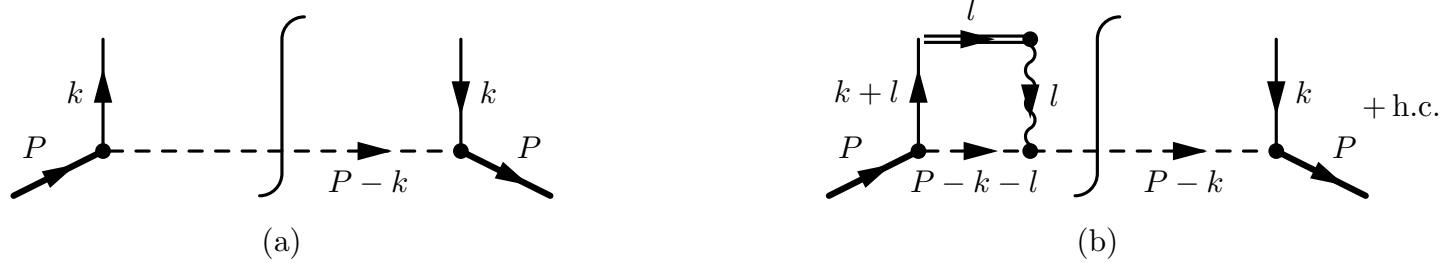
(Burkardt, 2005 / Meißner, Metz, Goeke, 2007)

- Information on chiral odd GPDs (Pasquini, Pincetti, Boffi, 2005 / QCDSF, 2007)
- Implies: $h_1^{\perp u/p} < 0$ $h_1^{\perp d/p} < 0$
- Agrees, e.g., with spectator model calculations
(Gamberg, Goldstein, Schlegel, 2007 / Bacchetta, Conti, Radici, 2008 / etc.)

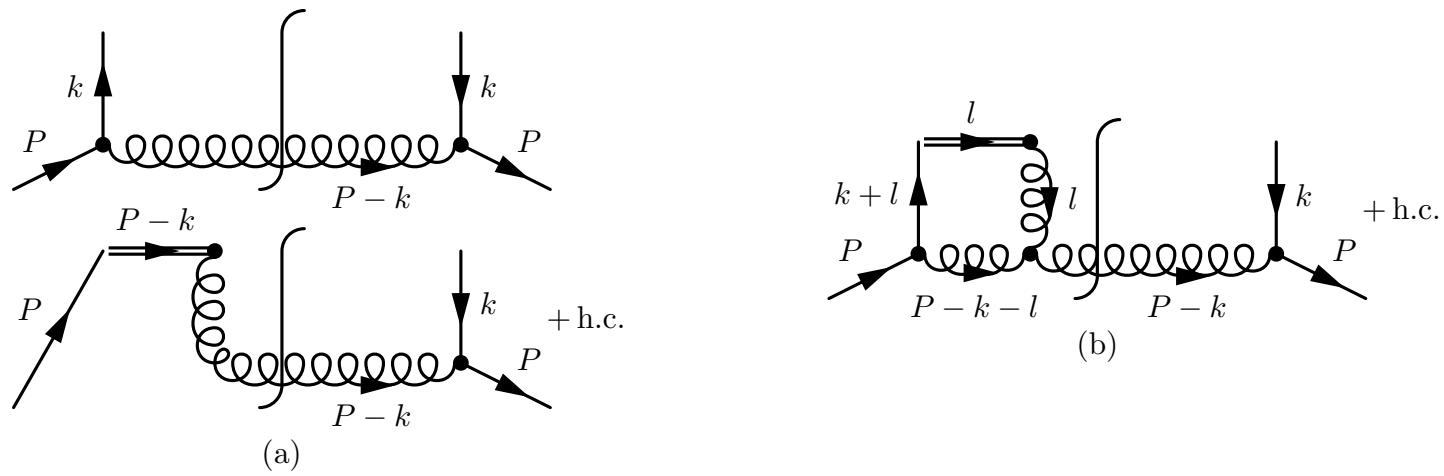
- Relation for $h_{1T}^{\perp q}$ expected to be different

Model results, continued

- Scalar diquark spectator model of the nucleon



- Quark target model in QCD



- Moments of GPDs and TMDs

$$X^{(n)}(x) = \frac{1}{2M^2} \int d^2 \vec{\Delta}_T \left(\frac{\vec{\Delta}_T^2}{2M^2} \right)^{n-1} X\left(x, 0, -\frac{\vec{\Delta}_T^2}{(1-x)^2}\right)$$

$$Y^{(n)}(x) = \int d^2 \vec{k}_T \left(\frac{\vec{k}_T^2}{2M^2} \right)^n Y(x, \vec{k}_T^2)$$

- Relations of second type

$$f_{1T}^{\perp q(n)}(x) = H_2(n) \frac{1}{1-x} E^{q(n)}(x) \quad (0 \leq n \leq 1)$$

→ $H_2(n)$ depends on model

→ Formula holds for all the relations of second type

→ Particular cases

$$f_{1T}^{\perp q(0)}(x) = \frac{\pi e_q e_s}{48(1-x)} E^q(x, 0, 0) \quad (\text{Lu, Schmidt, 2006})$$

$$f_{1T}^{\perp q(1)}(x) = \frac{e_q e_s}{4(2\pi)^2 (1-x)} E^{q(1)}(x) \quad (\text{Burkardt, Hwang, 2003})$$

- Relations of third type

$$h_{1T}^{\perp q(n)}(x) = H_3(n) \frac{1}{(1-x)^2} \tilde{H}_T^{q(n)}(x) \quad (0 \leq n \leq 1)$$

- $H_3(n)$ is the same in both models
- Formula holds for all the relations of third type
- Particular cases

$$h_{1T}^{\perp q(0)}(x) = \int d^2\vec{k}_T h_{1T}^{\perp q}(x, \vec{k}_T^2) = \frac{3}{(1-x)^2} \tilde{H}_T^q(x, 0, 0)$$

$$h_{1T}^{\perp q(1)}(x) = \int d^2\vec{k}_T \frac{\vec{k}_T^2}{2M^2} h_{1T}^{\perp q}(x, \vec{k}_T^2) = \int d^2\vec{b}_T \frac{\vec{b}_T^2}{2M^2} 2 \left(\tilde{\mathcal{H}}_T^q(x, \vec{b}_T^2) \right)''$$

- No immediate evidence for breakdown of relations of third type

- Relation of fourth type

- Trivially satisfied because

$$h_{1T}^{\perp g} = \tilde{\mathcal{H}}_T^g = 0$$

GTMD analysis

- GTMD-correlator (e.g., Belitsky, Ji, Radyushkin, Yuan, 2003, 2005)

$$W^q = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2 \vec{z}_T}{(2\pi)^2} e^{ik \cdot z} \langle p'; \lambda' | \bar{\psi}\left(-\frac{z}{2}\right) \gamma^+ \mathcal{W}_{GTMD} \psi\left(\frac{z}{2}\right) | p; \lambda \rangle \Big|_{z^+=0}$$

- Projection onto GPDs and TMDs

$$\begin{aligned} F^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik \cdot z} \langle p'; \lambda' | \bar{\psi}\left(-\frac{z}{2}\right) \gamma^+ \mathcal{W}_{GPD} \psi\left(\frac{z}{2}\right) | p; \lambda \rangle \Big|_{z^+=z_T=0} \\ &= \int d^2 \vec{k}_T W^q \end{aligned}$$

$$\begin{aligned} \Phi^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2 \vec{z}_T}{(2\pi)^2} e^{ik \cdot z} \langle p; \lambda' | \bar{\psi}\left(-\frac{z}{2}\right) \gamma^+ \mathcal{W}_{TMD} \psi\left(\frac{z}{2}\right) | p; \lambda \rangle \Big|_{z^+=0} \\ &= W^q \Big|_{\Delta=0} \end{aligned}$$

- GPDs and TMDs appear as certain limits of GTMDs (mother distributions)
 → Which GPDs and TMDs have the same mother distributions ?

- Parameterization of GTMD-correlator

Example:

$$W^q = \frac{1}{2M} \bar{u}(p', \lambda') \left[F_{1,1} + \frac{i\sigma^{i+} k_T^i}{P^+} F_{1,2} + \frac{i\sigma^{i+} \Delta_T^i}{P^+} F_{1,3} + \frac{i\sigma^{ij} k_T^i \Delta_T^j}{M^2} F_{1,4} \right] u(p, \lambda)$$

→ GTMDs are complex functions: $F_{1,n} = F_{1,n}^e + iF_{1,n}^o$

- Implications for potential nontrivial relations

– Relations of second type

$$\begin{aligned} E(x, 0, \vec{\Delta}_T^2) &= \int d^2 \vec{k}_T \left[-F_{1,1}^e + 2 \left(\frac{\vec{k}_T \cdot \vec{\Delta}_T}{\vec{\Delta}_T^2} F_{1,2}^e + F_{1,3}^e \right) \right] \\ f_{1T}^\perp(x, \vec{k}_T^2) &= -F_{1,2}^o(x, 0, \vec{k}_T^2, 0, 0) \end{aligned}$$

→ No model-independent nontrivial relation between E and f_{1T}^\perp possible

→ Relation in spectator model due to simplicity of the model

→ No information on numerical violation of relation

→ Likewise for nontrivial relation involving h_1^\perp

- Relation of third type

$$\begin{aligned}\tilde{H}_T(x, 0, \vec{\Delta}_T^2) &= \int d^2\vec{k}_T \left[\left(\frac{\vec{k}_T \cdot \vec{\Delta}_T}{\vec{\Delta}_T^2} H_{1,1}^e + H_{1,2}^e \right) \right. \\ &\quad \left. - 2 \left(\frac{2(\vec{k}_T \cdot \vec{\Delta}_T)^2 - \vec{k}_T^2 \vec{\Delta}_T^2}{(\vec{\Delta}_T^2)^2} H_{1,4}^e + \frac{\vec{k}_T \cdot \vec{\Delta}_T}{\vec{\Delta}_T^2} H_{1,5}^e + H_{1,6}^e \right) \right] \\ h_{1T}^\perp(x, \vec{k}_T^2) &= H_{1,4}^e(x, 0, \vec{k}_T^2, 0, 0)\end{aligned}$$

- No model-independent nontrivial relation between \tilde{H}_T and h_{1T}^\perp possible
 → Relation in spectator model

$$\begin{aligned}H_{1,1}^e(\xi = 0) &= H_{1,5}^e(\xi = 0) = 0 \\ 2\tilde{H}_{1,2}^e(x) - 4\tilde{H}_{1,6}^e(x) &= (1-x)^2 \tilde{H}_{1,4}^e(x)\end{aligned}$$

- Relation in light-front constituent quark model
 (Pasquini, Cazzaniga, Bozzo, 2008, 2009)

$$\begin{aligned}\int d^2\vec{k}_T h_{1T}^\perp(x, \vec{k}_T^2) &= \frac{2}{(1-x)^2} \tilde{H}_T(x, 0, 0) \quad \text{instead of} \\ \int d^2\vec{k}_T h_{1T}^\perp(x, \vec{k}_T^2) &= \frac{3}{(1-x)^2} \tilde{H}_T(x, 0, 0) \quad \text{in spectator model}\end{aligned}$$

Summary

- Various nontrivial relations between GPDs and TMDs established in low order spectator model calculations
- Neglecting certain diagrams relations of second type also hold in higher orders in certain spectator models (Gamberg, Schlegel, 2009)
- Relation between E^q and $f_{1T}^{\perp q}$ nicely agrees with phenomenology
- Relations (of second type) break down in spectator models in higher orders
- GTMD analysis implies that no model-independent nontrivial relations possible (analysis also for subleading twist)
- So far not much definite known about the numerical violation of the relations